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A Systematic Study on Recent Evolutionary Algorithms for Multi-Modal Multi-Objective Optimization

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Abstract:

The real-world optimization problems are inherently multi-modal and multi-objective (MMMO), such as bio-medical imaging, automotive engine design, plant identification in control systems, inference engine design, etc. This is mainly because of the acute diversity and diffusion of solutions in Pareto space and the multi-modality of the solution set. Therefore, finding the optimal solution for MMMO is a pressing need in the literature. It is evident from the literature that evolutionary Algorithms (EA) are the best candidates for solving MO problems. However, due to massive variants of single-objective EAs (SOEA) and Multi-objective EA (MOEA), the advocacy of the best EA for the MMMO problem needs to be improved in the literature. This work has presented a comprehensive study on the recent variants of EA to solve single-objective and multi-objective optimization problems. In addition, the Uni-modal and multi-modal problems are also considered with the respective EA. In addition to synthesizing the previous research, the work also highlights the knowledge gaps concerning the evolutionary algorithms for MMMO situations and offers a systematic framework for the additional study. Moreover, the work also highlights the usefulness and the outcomes of these algorithms in real-time situations, increasing their applicability in solving challenging optimization problems in various fields.

Keywords: *Multi-modal and multi-objective problems, evolutionary algorithms, single objective optimization problems, Single Objective Evolutionary Algorithms (SOEA), Multi Objective Evolutionary Algorithms (MOEA), and optimization methods.*

I. INTRODUCTION

A. Optimization:

Optimization is a method for finding optimum solutions to real-world problems. The optimum solution is the maximization or minimization of the fitness or cost, respectively. However, the solution must satisfy the constraints and bounds [1]. Optimization can be categorized as constrained and unconstrained optimization [2]. Likewise, the problem could be single or multi-objective about the objective [3]. The optimization domain encompasses the optimum solution of single objective optimization (SOO) and MOO (MOO).

B. Single Objective Optimization (SOO) and Multi-Objective Optimization (MOO)

The purpose of SOO is to find optimal solutions for single-objective problems. Specifically, it is the procedure to determine and evolve the finest answer in the entire search space to attain a single objective [4]. Various OAs, specifically gradient descent, simulated annealing, and evolutionary algorithms, can be employed to solve these optimization problems. Some examples of SOO problems include finding the minimum cost of a manufacturing procedure, maximizing the profit of a business, and minimizing the time required to complete a task. The scenario looks beneficial. However, it is found to be constrained while dealing with optimization problems with multiple objectives. Real-world problems often involve numerous objectives, and finding a solution which optimizes all objectives simultaneously can be challenging. In these cases, it may be necessary to use multi-objective MOO methods to find an answer which balances conflicting objectives. MOO aims to identify solutions that balance the various objective functions. This set of solutions is known as the "PF (PF)," it represents the best possible solutions that can be achieved without sacrificing one objective function for the sake of another. Multiple techniques and algorithms are available for MOO, specifically evolutionary algorithms, linear programming, and gradient descent, to name a few. These techniques are widely used in economics, computer science and engineering to determine optimal solutions to problems with conflicting objectives. Technically, in the literature, it is referred to as MOO [5]. In MOO, multiple objective functions are optimized concurrently [5]. The scope of MOO includes but is not limited to computing [6], engineering [7], economics [8], and logistics [9], which have multiple objectives.

C. Constrained and Un-Constrained Optimization (CO & UCO)

In addition, the optimization domain can also be categorized as constrained and unconstrained optimization [10]. In the constrained optimization, the domain of optimal solution has a clear bound. The expected solution must comply with the limits of the bounds [11]. These constraints must be satisfied with the solution, and they can be either equality constraints or inequality constraints. Equality constraints specify which relationship must hold exactly, while inequality constraints specify which one must hold at least as strictly as specified. For example, an equality constraint might be where the sum of decision variables must equal a specific value, while an inequality constraint might be when a decision variable must be greater than or equal to a particular value. Many different methods and algorithms may be used for constrained optimization, conditional to definite physical characteristics of the problem and the desired solution. Some standard techniques include gradient descent, linear programming, and quadratic programming. Constrained optimization is commonly used in various fields, specifically engineering, economics, and computer science, for discovering the most effective resolutions to multiple challenges, including resource allocation, scheduling, and control systems. As Shown in Figure 1. The left grey line shows the lower bound and the right line illustrates the upper bound of constraints. The set of possible solutions is mapped as a red curve. The solutions between the lower and upper bound are termed feasible solutions. At the same time, the set of solutions outside the bounds are termed infeasible solutions. Point A on the curve depicts the global minima / optimal solution.

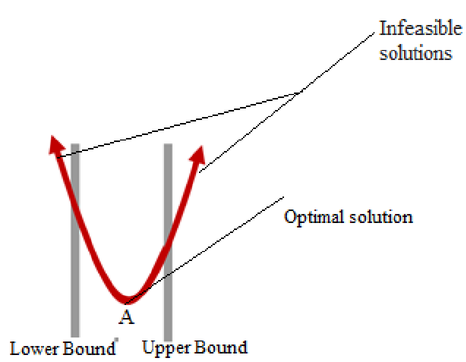

Figure 1: Constrained optimization

The un-constrained optimization problems need more flexibility in solution bounds. In un-constrained optimization, more weightage is given to meet the objective compared to satisfying the constraints [12]. Unconstrained optimization is finding the optimal solution to the problem without any constraints. Stated differently, unconstrained optimization aims to determine the optimal values for decision variables that either maximize or minimize the objective function, devoid of any constraints or limitations on the solutions. Many different methods and algorithms may be used for unconstrained optimization, conditional to distinctive features of the problem and the desired solution. Some standard techniques include gradient descent, and the method of steepest descent. Unconstrained optimization is widely utilized in several domains like economics, computer science and engineering to determine optimal solutions to various problems, including resource allocation, scheduling, and control systems. It is generally more straightforward to solve an unconstrained optimization problem than a constrained optimization problem because we do not need to worry about satisfying any constraints on the solution. However, in many real-world problems, it is frequently necessary to ponder constraints for finding a meaningful and feasible answer.

D. Single Objective Evolutionary Algorithms (SOEA)

A SOEA is an optimization method which uses evolutionary principles to find optimal solutions to problems with a single objective function [13]. However, the primary aim of this optimization is to identify the values of decision variables that either maximize or minimize the objective function without considering any supplementary objectives. Many variations of SOEA can be applied to various optimization problems [14]. For solving a single objective optimization problem using an evolutionary algorithm, we need to define an objective function representing the goal or target of the optimization problem and a set of decision variables representing the inputs or choices we can make to influence the objective function. We also need to define the parameters of the evolutionary algorithm, precisely the population size, selection criteria, and mutation rate. An evolutionary algorithm is initiated by generating a population comprising potential solutions assessed based on an objective function. The best-performing solutions are then selected to produce the next generation through crossover and mutation procedures. This procedure is reiterated till an optimal solution is identified or the criterion is met. Evolutionary algorithms focusing on a single objective are widely employed when an optimization problem has many variables, or the objective function is too complex to optimize directly. SOEAs are often preferred over traditional optimization methods because of their ability to search for optimal solutions more efficiently and flexibly. However, they can be slower and require more computation than other methods, and they may only sometimes find the global optimum of the objective function.

E. Multi-Objective Evolutionary Algorithm (MOEA)

Multi-objective EA is used when we have more than one objective function, which needs to be optimized simultaneously to return multiple optimal solutions (referred to as Pareto Optimal PO solutions) for fulfilling each objective function [15]. The outcome is obtained from combining all these solutions as a curve. It is also called PO front, or, we can say, POF. An MOEA is an optimization method that uses principles of natural evolution, specifically selection and reproduction, to search for optimal solutions to problems with multiple conflicting objectives [16]. In other words, the goal of an MOEA is to find a set of solutions that satisfy various objectives simultaneously rather than just one objective, as in single objective optimization. In a MOEA, a population of candidate solutions is initialized and then evolves through generations. At each generation, solutions in the population are evaluated according to multiple objectives, and subsequently, the most superior solutions are chosen to generate the succeeding generation of solutions. The technique is continued until PF is located or a predefined stopping threshold is fulfilled. MOEAs come in various forms, including NSGA-II, SPEA2, and MOEA/D. These approaches apply to a broad range of optimization issues, including continuous optimization, discrete optimization, and MOO. MOEAs are often preferred over traditional optimization methods because they can search for optimal solutions more efficiently and flexibly. However, they can be slower and require more computation than other methods, and they may only sometimes find global PF of objectives.

F. Single Modal Optimization (SMO) and Multi-Modal Optimization (MMO)

SMO refers to optimizing a single mode or component of a system to achieve a desired performance outcome [17]. This is in contrast to MMO, which involves optimizing multiple components or modes of a system simultaneously to achieve an overall optimal result. SMO is typically performed in cases where the objective is to optimize one specific aspect of a system, for example, the speed of a vehicle or the energy efficiency of a building. Multi-modal optimization occurs when the problem has multiple optimal solutions for the problem. Some solutions act as global optimal, containing the same objective function, and some act as local optimal, containing different objective solutions. We either find all possible optimal solutions or find the most ones. MMO refers to finding the optimal solution to a problem with multiple local optima, or "modes," rather than just a single global optimum. In other words, an MMO problem is one in which numerous distinct solutions are optimal within their respective regions of solution space. Still, these solutions are only globally optimal across some of the space. MMO problems can be challenging because traditional optimization methods, specifically gradient descent are designed to find the global optimum. Still, they may encounter obstacles in the form of local optima, hindering the discovery of alternative global optima [18]. To solve an MMO problem, it is often necessary to use specialized methods to explore solution space more thoroughly and avoid getting stuck in local optima. Some methods commonly used for MMO include evolutionary algorithms, simulated annealing, and parallel tempering. MMO is frequently encountered in various fields, specifically engineering, economics, and computer science, and it is often relevant in problems involving the design of complex systems, optimization of financial portfolios, and the selection of parameter values in machine learning models.

G. Single Modal Single Objective (SMSO)

This approach is used when a single objective function is fulfilled to obtain the best solution for a specific problem. It denotes the procedure of enabling the pursuit of the most optimal problem-solving approaches with a single objective function with a single global optimum [19]. In other words, the single-modal single objective optimization problem is one in which there is a unique best solution, and all other solutions are worse in some way. Many different methods and algorithms may be used for single-modal, single-objective optimization, depending on the specific characteristics of the problem and the desired solution. Some standard techniques include gradient descent, and the method of steepest descent. Single-modal single objective optimization is commonly used in various fields, specifically engineering, economics, and computer science, for discovering the most effective resolutions to multiple challenges, including resource allocation, scheduling, and control systems. It is generally more straightforward to solve single-modal single-objective optimization problems than an MMO problem because it doesn't have to worry about getting stuck in a local optimum and can focus on finding the global optimum. However, when discussing real-world issues, it may be necessary to contemplate multiple objectives for identifying a meaningful and feasible solution.

H. Single Modal Multi-Objective (SMMO)

This approach is used when multiple objective functions are fulfilled to find solutions for a specific problem [20]. This denotes finding the optimal solution to the problem with numerous conflicting objectives with a single global optimum. In other words, a single modal MOO problem is one in which there is a single "peak" or "valley" in objective space, and optimal solutions lie on PF, representing the offsets between the conflicting objectives.

Many different methods and algorithms may be used for single-modal MOO, depending on the specific characteristics of the problem and the desired solution. Some standard techniques include evolutionary algorithms, multi-objective linear programming, and goal programming. Single-modal MOO is commonly used in various fields, specifically engineering, economics, and computer science, for discovering the most effective resolutions to multiple challenges, including resource allocation, scheduling, and control systems. Solving a single modal MOO problem can be more challenging than a single objective optimization problem because multiple conflicting objectives may be considered. However, in some cases, it may be necessary to consider numerous objectives to find a meaningful and feasible solution.

I. Multi-Modal Single Objective (MMSO)

This approach is used when multiple objective functions are fulfilled to obtain the best solution for a specific problem. This denotes finding the optimal solution to the problem with a single objective function with multiple local optima rather than a single global optimum. In this approach, there are several different "peaks" or "valleys" in the objective function, and the optimal solution depends on which peak or valley is chosen [21]. Multi-modal single-objective optimization problems can be challenging to solve because traditional optimization methods, specifically gradient descent are typically designed to find the global optimum of a single-modal objective function. These methods may get stuck in a local optimum and need help finding the global optimum. Many approaches exist for solving multi-modal single-objective optimization problems.

J. Multi-Modal Multi-Objective (MO)

MMMO refers to finding the optimal solution to the problem with multiple conflicting objectives, where the objective function has numerous local optima rather than a single global optimum. In other words, an MO problem is one in which there are several different "peaks" or "valleys" in objective space, and optimal solutions are the ones which satisfy multiple objectives simultaneously and lie on or near one of the local optima [22]. Optimal solutions in MOO are often represented as a set of points in objective space. These points represent the offsets between the conflicting objectives, and any solution that is not on PF can be dominated by at least one other solution on the front. MMMO problems can be particularly challenging to solve because traditional optimization methods, specifically gradient descent, are typically designed to find the global optimum of a single-modal objective function. These methods may get stuck in a local optimum and need help finding the global optimum. Several approaches could be utilized to solve MMMO problems, including evolutionary algorithms, multi-objective linear programming, and goal programming. These methods are commonly used in fields, specifically engineering, economics, and computer science, for discovering the most effective resolutions to various challenges, specifically resource allocation, scheduling, and control systems. Solving MO problems can be particularly challenging because we must simultaneously consider multiple conflicting objectives and local optima.

K. Real-World Examples of MO Multi-Modal Multi-Objective

There are many real-world examples of MO, including:

Resource Allocation: In many organizations, there are often multiple conflicting objectives when allocating resources, specifically maximizing profits, minimizing costs, and maximizing customer satisfaction [23]. An MO problem may be wielded to find the optimal allocation of resources that simultaneously satisfies all of these objectives.

Scheduling: In manufacturing and logistics, there are often multiple conflicting objectives when scheduling tasks, specifically minimizing production time, minimizing costs, and maximizing quality. An MO problem may be used to find an optimal schedule that satisfies these objectives.

Control systems: In control systems, there are often multiple conflicting objectives when designing the control system, explicitly minimizing the control effort, minimizing the control error, and maximizing the system stability [24]. An MMMO problem may be wielded to find an optimal control system design that satisfies all these objectives simultaneously.

Portfolio optimization: In finance, there are often multiple conflicting objectives when designing an investment portfolio, explicitly maximizing returns, minimizing risk, and maximizing diversification. A MMMO problem may be wielded to find an optimal portfolio that satisfies all these objectives simultaneously.

Supply chain optimization: In supply chain management, there are often multiple conflicting objectives when designing the supply chain, explicitly minimizing costs, maximizing efficiency, and minimizing environmental impact. A MMMO problem may be wielded to find an optimal supply chain design that satisfies all these objectives simultaneously.

II. LITERATURE REVIEW & GAP ANALYSIS

This section will present the recent literature (mainly from the last five years) on the research contribution to solving the multi-modal multi-objective optimization problems using single-objective and multi-objective evolutionary algorithms. A careful selection of the literature was established, and most of the research papers cited were from IEEE Transactions and high-impact journals. In addition, the research published in the Top Tears conference has also been included in this literature survey. Table 1 illustrates the list of algorithms for single Objective Optimization, and Table 2 refers to the list of Multi-Objective optimization algorithms covered in this literature review. In the subsequent section, each algorithm's principle functionality, strength and corresponding limitation are discussed at length.

Table 1: Single Objective Optimization Algorithm

Single Objective Optimization Algorithm			
S.No	**Abbreviation**	**Full name**	**Ref**
1.	ABC	Artificial bee colony algorithm	[25]
2.	ACO	Ant colony optimization	[27]
3.	Adam	Adaptive moment estimation	[28]
4.	BFGS	A quasi-Newton method proposed by Broyden, Fletcher, Goldfarb, and Shanno	[35]
5.	BSPGA	Binary space partition tree based genetic algorithm	[37]
6.	CMA-ES	Covariance matrix adaptation evolution strategy	[42]
7.	CSO	Competitive swarm optimizer	[48]
8.	DE	Differential evolution	[52]
9.	EGO	Efficient global optimization	[64]
10.	FEP	Fast evolutionary programming	[69]
11.	FRCG	Fletcher-Reeves conjugate gradient	[71]
12.	FROFI	Feasibility rule with the incorporation of objective function information	[72]
13.	GA	Genetic algorithm	[73]
14.	IMODE	Improved multi-operator differential evolution	[87]
15.	NelderMead	The Nelder-Mead algorithm	[133]
16.	OFA	Optimal foraging algorithm	[142]
17.	PSO	Particle swarm optimization	[155]
18.	RMSProp	Root mean square propagation	[157]
19.	SA	Simulated annealing	[166]
20.	SACC-EAM-II	Surrogate-assisted cooperative co-evolutionary algorithm of Minamo	[167]
21.	SACOSO	Surrogate-assisted cooperative swarm optimization	[168]
22.	SADE-Sammon	Sammon mapping assisted differential evolution	[169]
23.	SAMSO	Multiswarm-assisted expensive optimization	[170]
24.	SQP	Sequential quadratic programming	[180]

Table 2 Multi-Objective Optimization Algorithm

S.No	Abbreviation	Full name	Re
25.	AB-SAEA	Adaptive Bayesian based surrogate-assisted evolutionary algorithm	[26]
26.	AGE-II	Approximation-guided evolutionary multi-objective algorithm II	[29]
27.	AGE-MOEA	Adaptive geometry estimation-based many-objective evolutionary algorithm	[30]
28.	A-NSGA-III	Adaptive NSGA-III	[31]
29.	AR-MOEA	Adaptive reference points based multi-objective evolutionary algorithm	[32]
30.	BCE-IBEA	Bi-criterion evolution based IBEA	[33]
31.	BCE-MOEA/D	Bi-criterion evolution based MOEA/D	[34]
32.	BiGE	Bi-Goal evolution	[36]
33.	CA-MOEA	Clustering based adaptive multi-objective evolutionary algorithm	[38]
34.	CCGDE3	Cooperative coevolution GDE3	[39]
35.	CCMO	Coevolutionary constrained multi-objective optimization framework	[40]
36.	c-DPEA	Constrained dual-population evolutionary algorithm	[41]
37.	C-MOEA/D	Constraint-MOEA/D	[43]
38.	CMOEA-MS	Constrained multiobjective evolutionary algorithm with multiple stages	[44]
39.	CMOPSO	Competitive mechanism based multi-objective particle swarm optimizer	[45]
40.	CPS-MOEA	Classification and Pareto domination based multi- objective evolutionary	[46]
41.	CSEA	Classification based surrogate-assisted evolutionary algorithm	[47]
42.	C-TAEA	Two-archive evolutionary algorithm for constrained MOPs	[49]
43.	DAEA	Duplication analysis based evolutionary algorithm	[50]
44.	DCNSGA-III	Dynamic constrained NSGA-III	[51]
45.	DEA-GNG	Decomposition based evolutionary algorithm guided by growing neural gas	[53]
46.	DGEA	Direction guided evolutionary algorithm	[56]
47.	DMOEA-eC	Decomposition-based multi-objective evolutionary algorithm with the e-constraint framework	[54]
48.	dMOPSO	MOPSO based on decomposition	[55]
49.	DN-NSGA-II	Decision space based niching NSGA-II	[58]
50.	DSPCMDE	Dynamic selection preference-assisted constrained multiobjective differential evolution	[59]
51.	DWU	Dominance-weighted uniformity multi-objective evolutionary algorithm	[60]
52.	EAG-MOEA/D	External archive guided MOEA/D	[61]
53.	EDN-ARMOEA	Efficient dropout neural network based AR-MOEA	[62]
54.	EFR-RR	Ensemble fitness ranking with a ranking restriction scheme	[63]
55.	EIM-EGO	Expected improvement matrix based efficient global optimization	[65]

56.	e-MOEA	Epsilon multi-objective evolutionary algorithm	[66]
57.	EMyO/C	Evolutionary many-objective optimization algorithm with clustering-based	[67]
58.	ENS-MOEA/D	Ensemble of different neighborhood sizes based MOEA/D	[68]
59.	FDV	Fuzzy decision variable framework with various internal optimizers	[70]
60.	GDE3	Generalized differential evolution 3	[74]
61.	GFM-MOEA	Generic front modeling based multi-objective evolutionary algorithm	[75]
62.	GLMO	Grouped and linked mutation operator algorithm	[76]
63.	g-NSGA-II	g-dominance based NSGA-II	[77]
64.	GrEA	Grid-based evolutionary algorithm	[78]
65.	HeE-MOEA	Multiobjective evolutionary algorithm with heterogeneous ensemble based infill criterion	[79]
66.	hpaEA	Hyperplane assisted evolutionary algorithm	[80]
67.	HypE	Hypervolume estimation algorithm	[81]
68.	IBEA	Indicator-based evolutionary algorithm	[82]
69.	ICMA	Indicator based constrained multi-objective algorithm	[83]
70.	I-DBEA	Improved decomposition-based evolutionary algorithm	[84]
71.	IM-MOEA	Inverse modeling based multiobjective evolutionary algorithm	[85]
72.	IM-MOEA/D	Inverse modeling multiobjective evolutionary algorithm based on decomposition	[86]
73.	I-SIBEA	Interactive simple indicator-based evolutionary algorithm	[88]
74.	KnEA	Knee point driven evolutionary algorithm	[89]
75.	K-RVEA	Surrogate-assisted RVEA selection	[90]
83.	MaOEA-DDFC	Many-objective evolutionary algorithm based on directional diversity and favorable convergence	[98]
84.	MaOEA/IGD	IGD based many-objective evolutionary algorithm	[99]
85.	MaOEA/IT	Many-objective evolutionary algorithms based on an independent two-stage	[100]
86.	MaOEA-R&D	Many-objective evolutionary algorithm based on objective space reduction	[101]
87.	MMOPSO	MOPSO with multiple search strategies	[102]
88.	MO_Ring_ PSO_SCD	Multiobjective PSO using ring topology and special crowding distance	[103]
89.	MOCell	Cellular genetic algorithm	[104]
90.	MO-CMA	Multi-objective covariance matrix adaptation evolution strategy	[105]
91.	MOEA/D	Multiobjective evolutionary algorithm based on decomposition	[106]
92.	MOEA/D-AWA	MOEA/D with covariance matrix adaptation evolution strategy	[107]
93.	MOEA/D- CMA	MOEA/D with covariance matrix adaptation evolution strategy	[108]
94.	MOEA/DD	Many-objective evolutionary algorithm based on dominance and decomposition	[109]
95.	MOEA/D-DAE	MOEA/D with detect-and-escape strategy	[110]
96.	MOEA/D-DE	MOEA/D based on differential evolution	[111]
97.	MOEA/D-DRA	MOEA/D with dynamical resource allocation	[112]
98.	MOEA/D-DU	MOEA/D with a distance based updating strategy	[113]
99.	MOEA/D-DYTS	MOEA/D with fitness-rate-rank-based multiarmed bandit	[114]
100.	MOEA/D-EGO	MOEA/D with efficient global optimization	[115]
101.	MOEA/D-FRRMAB	MOEA/D with fitness-rate-rank-based multiarmed bandit	[116]
102.	MOEA/D- M2M	MOEA/D based on MOP to MOP	[117]
103.	MOEA/D-MRDL	MOEA/D with maximum relative diversity loss	[118]
104.	MOEA/D-PaS	MOEA/D with Pareto adaptive scalarizing approximation	[119]
105.	MOEA/D-STM	MOEA/D with stable matching	[120]
106.	MOEA/D-UR	MOEA/D with update when required	[121]
107.	MOEA/D-URAW	MOEA/D with uniform randomly adaptive weights	[122]

III. RESEARCH GAP

Multi-modal MOO (MMMO) using evolutionary algorithms (EA) is a growing field of research. Despite the progress made, there are still several gaps in the study of MO using EAs. Some of these include:

Scalability: MMMO problems often involve many variables and objectives, making it challenging for EAs to find the global optimum. Researchers are still exploring ways to make MMMO algorithms scalable to more significant problems.

Diversity preservation: Maintaining various methods or approaches is crucial for MMMO algorithms, as it helps to ensure that all optimal solutions are found. However, preserving diversity in the face of convergence can be challenging.

Handling constraints: Incorporating constraints into MO algorithms is often tricky, and existing methods are only sometimes practical.

Hybridization with other OT: EAs can be combined with other OT, specifically gradient-based methods, to improve the performance of MMMO algorithms. However, there is still much room for improvement in this area.

Improved EA: There is a need to design an improved EAs for Multi-Modal Multi-Objective Problem

IV. CONCLUSION

The research paper has provided a systematic study on recent evolutionary algorithms for multi-modal multi-objective optimization (MMMO). The work has addressed the pressing need in various real-world applications where optimization problems exhibit multi-modal and multi-objective characteristics. The study has provided a comprehensive understating of the approaches employed in tackling the MMMO problems by analyzing both single objective and multi-objective evolutionary algorithms. The paper also outlined the challenges posed by multi-objective optimization and introduced the concept of Pareto optimal solutions. Additionally, the distinction between single-modal and multi-modal optimization was elucidated, highlighting the complexity introduced by multiple optimal solutions. Different types of optimization problems have been considered including the single-modal single-objective, single-modal multi-objective, multi-modal single-objective, and multi-modal multi-objective. Real-world examples were provided to illustrate the relevance of MMMO problems in diverse fields such as resource allocation, scheduling, control systems, portfolio optimization, and supply chain management. The paper also investigated into recent research contributions in solving MMMO problems using evolutionary algorithms. The tables listing single-objective and multi-objective optimization algorithms facilitated a comprehensive understanding of the landscape in this research domain. To sum up, the study conducted serves as a valuable resource for researchers and practitioners in the field of optimization and artificial intelligence, as it provides insights into the latest advancements in evolutionary algorithms for MMMO problems. In addition to this, this work exclusively contributes to the ongoing efforts to address complex optimization challenges in real-world applications by highlighting the principles, strengths, and limitations of various approaches.

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