Improved Technique: An Alternative Method of Nodal Analysis

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***Abstract*- Electric circuits plays dynamic role in each aspects of electrical engineering. An electrical engineer talent is to study in what manner circuits are split up into simpler parts. Though, breaking up hitches into lesser stages is the core of engineering. This research offers a sample of engineering approach to problem solving in modest and effective way. In Circuit analysis, means working out voltages and currents in each component. Node Voltage method is an organized scheme in investigating a circuit. Kirchhoff's Current Law (KCL) is precondition for nodal analysis, it selects node voltage as circuit parameter that supports in minimizing the number of equations that makes the design and calculation easier. This paper reports improved method of nodal analysis that computes node voltage based on the information of Ohm's Law only. This is an easy-going technique, much simpler, carries lesser amount of time, reduces the circuit complications and keeps the calculation easier and informal as compare to formal Nodal analysis.**

Keywords: Extra Node, Home Node, Kirchhoff's Current Law, Modified Equivalent Circuit, Nodal Analysis.

I. Introduction

Energy is present everywhere in nature in different forms but the most significant form of energy is electrical energy. In modern time every one is dependent on the use of electrical energy which is almost become a part of our life. An electric circuit plays a significant role in electrical engineering. It transmits power that is used for energy purposes like to run electric appliances, medical instruments etc. Several applications related to electrical circuits are observed in [1].

As the study says that mentioned in [2-8], there are numerous methods like Kirchhoff's Current Law (KCL), Kirchhoff's Voltages Law (KVL), Mesh analysis and Nodal analysis etc. to resolve electric circuit parameters (node voltage, element current or voltage).

According to the earlier researches elaborated in [9-10], Nodal analysis is a versatile procedure for examining circuits by node voltages.

In previous researches [11-12] it is discussed that; a node is an intersection point where two or more than two elements combine.

Aiming to avoid the complication in circuit analysis and to get rid from electrical rules formalities, such as KCL, we developed a new technique that will present a unique idea to get node voltages using only the basic knowledge of mathematics. DC circuits and Ohm's Law are the prerequisite for good understanding of this method as mentioned in [13].

It can be implemented in electric network designing, voltage distribution, power management and complex network calculations such as buildings, domestic electric wiring, industry and airplane etc.

II. GENERALIZED FORM

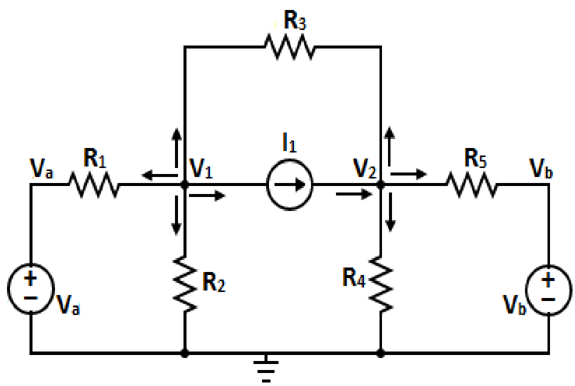

Fig. (1). A generalized circuit to find node voltages.

The circuit in Figure 1 consists of three elements; independent current source I, voltage source V and resistors R.

Step: 01 Firstly, find the LCM of all resistors present in the circuit. Let R_{LCM} be the LCM of the all resistances.

Step: 02 Next step is to find ratio of all resistors with R_{LCM} and is denoted as a, b, c, d, e and so on.

$$a = \frac{R_{LCM}}{R_1} \quad (1)$$

$$b = \frac{R_{LCM}}{R_2} \quad (2)$$

$$c = \frac{R_{LCM}}{R_3} \quad (3)$$

$$d = \frac{R_{LCM}}{R_4} \quad (4)$$

$$e = \frac{R_{LCM}}{R_5} \quad (5)$$

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$$z = \frac{R_{LCM}}{R_n} \quad (6)$$

If there is any independent current (I_1, I_2, I_3 …) then multiply R_{LCM} with it (it becomes a voltage source $IR = V$) and will be treated as independent voltage source V_{LCM} but its direction remains same and voltage source remains unchanged. If there are nth current sources then voltages will be

$$V_{LCM_1} = R_{LCM} I_1 \qquad (7)$$

$$V_{LCM_2} = R_{LCM} I_2 \qquad (8)$$

$$V_{LCM_3} = R_{LCM} I_3 \qquad (9)$$

$$V_{LCM_n} = R_{LCM} I_n \qquad (10)$$

Step 03: Finally, construct the equivalent circuit to make the calculation easier and this is the main step of our new approach as shown in figure 02.

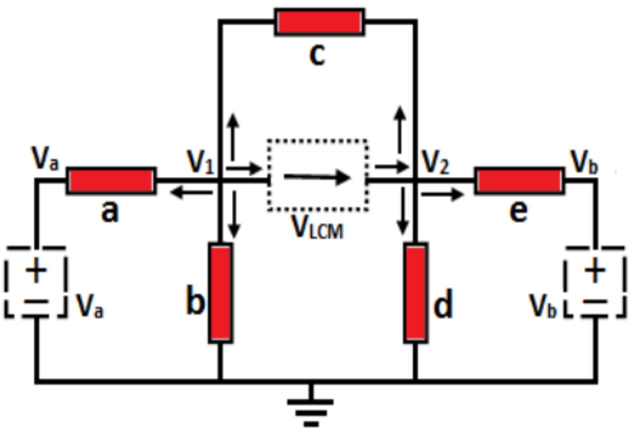

Fig. (2).: Equivalent circuit, the rectangular boxes show the resistance ratio a, b, c ... z and are not treated as resistor anymore.

To solve the equivalent circuit, calculate the equations for node voltage (V_1 & V_2).

For node voltage V_1 (Home Node)

Suppose V_1 is Home Node.

Here we are considering Home node as the node under observation and Extra node are the nodes connected to home node.

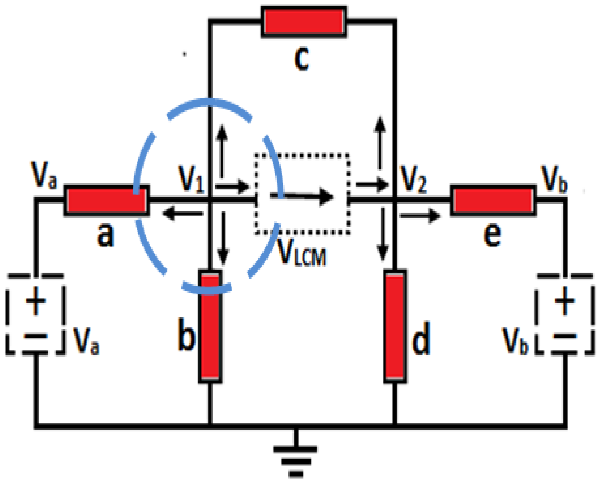

Fig. (3). Circuit representing resistance ratios connected to home node V_1.

On the left side of figure 03, take the sum of all resistor ratios connected to Home Node V_1 and multiply with V_1, similarly on right side multiply extra node (in this case V_a & V_2) to the resistors (if any) connected to it.

Current source is converted to voltage source as discussed earlier. According to this rule, If the direction of the current source is away from the home node, write it with home node values and if towards the home node, it will be written with extra node values.

$$(a+b+c)V_1 + V_{LCM} = (a)V_a + (c)V_2 \quad (11)$$

For node voltage V_2 (Home Node)

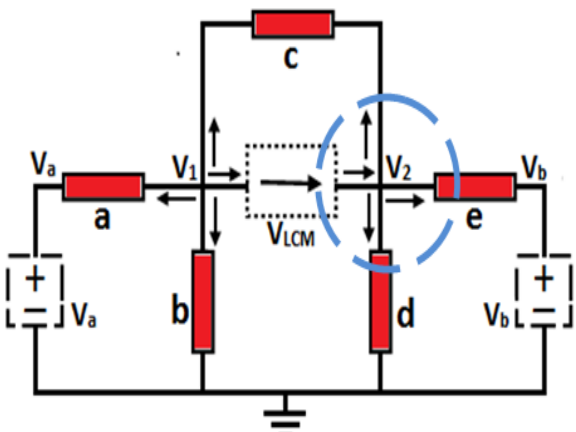

Fig. (4). Circuit representing resistance ratios connected to home node V_2.

Similarly, from figure 4, the equation for V_2 will be

$$(c + d + e)V_2 = (e)V_b + (c)V_1 + V_{LCM} \quad (12)$$

Now, the value of V_1 and V_2 can be find by using Inverse method, Substitution, Cramer's rule, Gauss Jordan Elimination method or by using calculator mentioned in [7-9].

III. NUMERICAL ANALYSIS

Below few examples discussed, shows the comparison of Nodal Analysis and Modified Nodal Analysis.

Example 01(A): Find the node voltages by using Nodal Analysis.

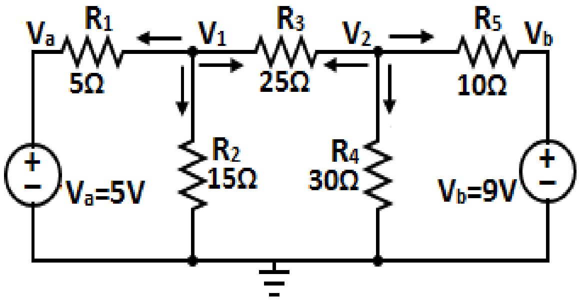

Fig. (5). Circuit without current source.

Solution:

At Home Node V_1:

According to KCL,

$$\frac{V_1 - V_a}{5} + \frac{V_1}{15} + \frac{V_1 - V_2}{25} = 0$$

$$\therefore V_a = 5V$$

$$23V_1 - 3V_2 = 75 \rightarrow (13)$$

At Home node V_2:

According to KCL,

$$\frac{V_2 - V_b}{10} + \frac{V_2}{30} + \frac{V_2 - V_1}{25} = 0$$

$$\therefore V_b = 9V$$

$$-6V_1 + 26V_2 = 135 \rightarrow (14)$$

Now by solving Equations (13) and (14), V_1 and V_2 can be find out as

$$V_1 = \mathbf{4.060V} \; and \; V_2 = \mathbf{6.129V}$$

Example 01(B): Find the node voltages by using Modified Nodal Analysis for the same circuit as in figure 05.

Solution:

Take LCM of all Resistors:

2	5,	10,	15,	25,	30
3	5,	5,	15,	25,	15
5	5,	5,	5,	25,	5
5	1,	1,	1,	5,	1
	1,	1,	1,	1,	1

Fig. (6). LCM of all resistors used in figure 05.

$$R_{LCM} = (2)(3)(5)(5)$$

$$R_{LCM} = \mathbf{150\Omega}.$$

Modified Equivalent Circuit:

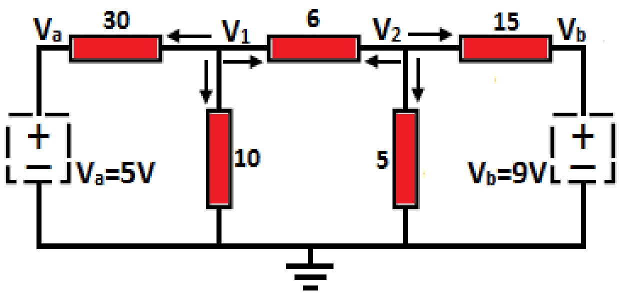

Fig. (7). Modified equivalent circuit of Fig (5).

At Home Node V_1:

$$(30 + 10 + 6)V_1 = 6V_2 + 30(V_a)$$

$$\therefore V_a = 5\text{V}$$

$$46V_1 - 6V_2 = 150 \;\rightarrow (15)$$

At Home Node V_2:

$$(6+5+15)V_2 = 6V_1 + 15V_b$$

$$\therefore V_b = 9\text{V}$$

$$-6V_1 + 26V_2 = 135 \;\rightarrow (16)$$

Now V_1 and V_2 can be achieved by solving Equations (15) and (16).

$$V_1 = \mathbf{4.060V} \; and \; V_2 = \mathbf{6.129V}$$

Example 02(A):

Find the node voltages by using Nodal Analysis.

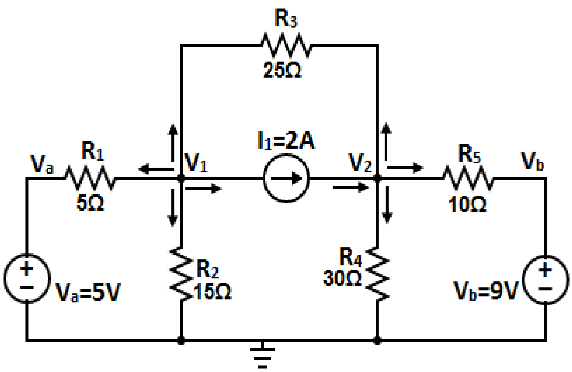

Fig. (8). Circuit including current source.

Solution:

At Home node V_1:

According to KCL:

$$\frac{V_1 - V_a}{5} + \frac{V_1}{15} + \frac{V_1 - V_2}{25} = -I_1$$

$$\therefore V_a = 5V \;\&\; I_1 = 2A$$

$$23V_1 - 3V_2 = -75 \;\rightarrow (17)$$

At Home node V_2:

As per KCL:

$$\frac{V_2 - V_b}{10} + \frac{V_2}{30} + \frac{V_2 - V_1}{25} = I_1$$

$$\therefore V_b = 9V \ \& \ I_1 = 2A$$

$$-6V_1 + 26V_2 = 435 \ \rightarrow (18)$$

Now V_1 and V_2 can be achieved by solving Equations (17) and (18).

$$V_1 = \mathbf{-1.112V} \ and \ V_2 = \mathbf{16.474V}.$$

Example 02(B): Find the node voltages by using Modified Nodal Analysis for the same circuit in Figure 08.
Solution:
LCM of all Resistors:

2	5,	10,	15,	25,	30
3	5,	5,	15,	25,	15
5	5,	5,	5,	25,	5
5	1,	1,	1,	5,	1
	1,	1,	1,	1,	1

Fig. (9). LCM of resistors used in Figure 8.

$$R_{LCM} = \mathbf{150\Omega}.$$

Modified Equivalent Circuit:

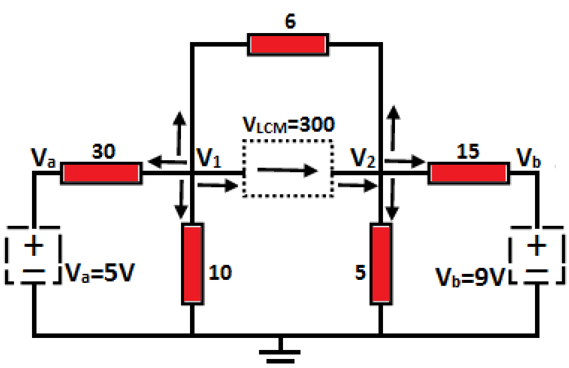

Fig. (10). Modified Equivalent circuit for Figure 8.

At Home node V_1:

$$(30 + 10 + 6)V_1 + V_{LCM} = 6V_2 + 30V_a$$

$$\therefore V_a = 5V \;\&\; V_{LCM} = 300V$$

$$46V_1 - 6V_2 = -150 \;\rightarrow (19)$$

At Home node V_2:

$$(6 + 5 + 15)V_2 = 6V_1 + 15V_b + V_{LCM}$$

$$\therefore V_b = 9V \;\&\; V_{LCM} = 300V$$

$$-6V_1 + 26V_2 = 435 \;\rightarrow (20)$$

Similarly, V_1 and V_2 can be achieved by solving Equations (19) and (20).

$$V_1 = \mathbf{-1.112V} \; and \; V_2 = \mathbf{16.474V}.$$

IV. RESULT AND DISCUSSIONS

Example No.	**Nodal Analysis**	**Modified Method**
1	$V_1 = 4.06V$ & $V_2 = 6.129V$	$V_1 = 4.060V$ & $V_2 = 6.129V$
2	$V_1 = -1.112V$ & $V_2 = 16.474V$	$V_1 = -1.112V$ & $V_2 = 16.474V$

Table 01: Comparison between Nodal Analysis and Modified method.

From the above analysis in Table 01, it can be clearly noticed that both methods providing same results and almost have same number of steps, but Modified method is easier to understand, consume less time than Nodal Analysis. This method provides direct equation without the knowing Kirchhoff's Current Law (KCL), only the knowledge of Least Common Multiple (LCM) and Ohm's Law is enough.

V. FUTURE IMPLEMENTATIONS

Modified method can be implemented on the super nodal circuits and it may be helpful for solving the circuits with dependent voltage or current sources. It can be applied to RC, RL and RLC circuits as well.

VI. CONCLUSION

Electric circuits play major role in our life. Many engineers, scientists have tried to make their solution easy; the efforts of Mesh and Nodal are unforgettable. But this modified analysis has presented another easy way to simplify electric circuit through LCM and Ohm's Law. So, it is finally concluded from the results that this method is more flexible and less time consuming as compare to Nodal Analysis.

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