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PERFORMANCE OF MODIFIED PROGRESSIVE MEAN CONTROL CHART WITH DIFFERENT CONSTANT VALUE

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***Abstract*: Any manufacturing process cannot produce exactly identical items. In all kinds of processes, a certain amount of variability is present, which causes such kind of variations in the products. A modified progressive mean (MPM) control chart has recently been proposed plotting the statistic which uses the present data as well as the past data. We used the modified progressive mean (MPM) control chart with a different value of constant K_0. The results showed an improved performance of the proposed chart for smaller, moderated and larger shifts in terms of power.**

*Keywords****: Run length, FIR, Progressive mean.***

I. INTRODUCTION

Quality has turned out to be a standout amongst the most imperative consumer judgment factors that choices within the conflicting issues. Manufacturing processes suffers from the problem of variability in the products. The variations are of two types: natural and unnatural variations. The natural variations are built-in random variations and a permanent part of the process which are due to equipment used, machinery operator and resources type etc. The other types of variations are called un-natural variations. The factors causing these types of variations can be detected and identified. These factors are non-random and large in magnitude. The process working under both natural and un-natural variations are said to be statistically out-of-control (Russell and Taylor, 1998).

The process can be controlled by continuous monitoring. This is done by taking samples on a fixed interval of time from the process and then checking them by some statistical tools. Statistical Process Control (SPC) is a famous collection of tools that are particularly used for process monitoring. It contains seven tools known to be magnificent seven and these are the cause and effect diagram, check sheet, Pareto chart, defect concentration diagram, Histogram, scatter diagram and control charts. Collectively, they are called SPC-tool kit. These seven control charts are the most famous and widely used to monitor the changes in location and distribution of the process. Control charts are basically the graphs which visually show that weather a process is in control or not. To know how we can decide process is in-control or not by just looking at the graph we have to take a look on design structure of the control charts. Generally, control charts have three parameters; Upper Control limit (UCL), Centre Line (CL) and Lower Control limit (LCL).

The average run length (ARL) is most famous measure of the performance of control charts. It is defined as an average number of samples we have to wait until a shift in the process mean, variance or any other parameter of interest is detected. Other performance measures are also based on ARL (i.e. Power and ATS etc.). In Shewhart-type control charts ($\bar{X}, S, S^2$) the distribution of ARL is geometric. So, if "p" is the probability of detecting a shift then the mean of geometric distribution is given by $1/p$.
But in the case of memory control charts, an assumption regarding a geometric random variable is violated which states that every event should be independent. So, in the case of these charts Run Length (RL) distribution is not

geometric. We find the distribution of RL and then compute the ARL for memory control charts. ARL_0 represents the in control ARL and ARL_1 represents the out-of-control ARL. In control charts, for a prefixed ARL_0, we try to minimize ARL_1 as much as possible using different techniques. There are two types of control charts: memoryless control charts and memory control charts. Memoryless control charts (Shewhart Type) are effective in detecting large shifts.

The two effective techniques: Cumulative Sum (CUSUM) and Exponentially Weighted Moving Average (EWMA) control chart are memoryless control charts used in the process of small shifts. These techniques are detailed below.

The Cumulative Sum (CUSUM) Control Chart: The CUSUM control chart was first introduced by [1] which is more efficient in detecting small shifts in the mean of the process. The (CUSUM) chart includes directly all the information with the cumulative sum of the deviation with the sequence of sample values from the target value are plotted. The (CUSUM) chart monitors the process average that the supported samples are taken from the process at given hours, days, weeks, months and etc.

Assume that the samples of sizes n ≥ 1 are collected, and the jth sample average is $\bar{X}$ and μ_0 is the target for the process mean, the cumulative sum control chart is defined as;"

$$\text{“}C_i = \sum_{j=1}^{i}(\bar{X} - \mu_0)\text{”}$$

Where the C_i and i are the sample numbers are called the cumulative sum of all samples that are included. From several samples, they combine the information. Let x_t be the *jth* value on the process and follow a normal distribution having mean μ_o and standard deviation σ_o if the process is in the control, also assume that the σ_o is known or a reliable estimate is presented.

In the applications of SPC, these assumptions are very consistent, in which situation the CUSUM is most useful.

The upper and lower limits are computed as:

$$C_i^+ = \max[\,0, x_i - (\mu_o + k)\; C_{i-1}^+]$$

$$C_i^- = \max[\,0, (\mu_o - k) - x_i + \; C_{i-1}^-]$$

Where the Initial value $\quad C_0^+ = C_0^- = 0$

Where μ_o represents the target mean and k is reference/slack value that is taken as partial of the shift we want to detect.

Exponentially Weighted Moving Average (EWMA) Control Chart: Similar to the CUSUM chart, the EWMA chart is useful in detecting small shifts in the process mean [2]. These charts are used to monitor the mean of a process based on samples taken from the process at given times (hours, shifts, days, weeks, months, etc.). EWMA chart gives exponentially subsiding weights in observations. This means that one can notice a decrease in weight with more remote observation. This control chart is also called a Geometric Moving Average (GMA). In the time series modeling and forecasting the EWMA chart is extensively used.

This control chart is defined as:

$$\text{“ } Z_t = \lambda X_t + (1-\lambda)Z_{t-1} \qquad 0 \le \lambda \le 1\text{”}$$

"The λ is constant, which can define the distance memory of EWMA weighting factor. The initial value requires for the first sample at $t = 1$, is the process mean, $Z_o = 0$. In some Problems, the starting value of the EWMA is the mean of initial data are also used, then $Z_o = \bar{X}$. The corresponding factor λ, in the calculation of EWMA statistic gives the rate when the older data are entered. A large value of λ provides less weight to old data and extra weight to current data; The small value of λ provides extra weight to old data, when λ = 1, then EWMA reduces to Shewhart chart indicating that only the most recent observations control the EWMA."

Let X_t are i.i.d. having common variance σ^2 then the variance of the control statistic Z_t is define as:

$$\sigma^2 = [\{ 1 - (1 - \lambda)^{2t} \}\lambda/(2 - \lambda)].\sigma x^2$$

The variance rapidly is to converge its asymptotic value,

$$\sigma zt^2 = \{\frac{\lambda}{(2-\lambda)}\}.\sigma x^2.$$

If λ is small. The EWMA control limits are,

$$LCL = Z_0 + L_{\sigma_{zt}}$$

$$CL = Z_0$$

$$UCL = Z_0 - L_{\sigma_{zt}}$$

The value of factor L is chosen from tables of [3]. Assumed that the process data are independently and normally distributed. According to other control techniques, the EWMA control techniques depends on the record of a quantities that are the representative of the procedure. When *t* becomes larger than the term $1 - (1 - \lambda)^{2t}$ gets closer to the unity. This means that, after some phases of time the control limits will become stable through the time i.e., parallel to the center line. The parameters of the EWMA chart are the multiple of σ, denoted by L in the control limits and the weighting factor λ. For ARL performance of this control chart it is possible to choose these parameters.
Many modifications in EWMA and CUSUM control charts have been suggested to improve their detection methods. The authors [4], [5] express that the Cumulative Sum control chart is considerably more effective than the Shewhart control charts, worried to little varieties in the normal. The authors [6] and[3], displayed the "EWMA" control charts as a decent decision to distinguish changes in little expansion in the process norm. [5] gave the idea about the "FIR" feature in CUSUM charts. Also, several procedures have been developed for the EWMA chart to increase its efficiency of quick detection of small shifts for different situations by different researchers including [7] [8] 10] etc.

Authors [9] presented a strategy for envisioning quality control diagrams dependent on their truthful execution over demonstrated in control and insane regions of parameter regard. Various amendments are available in literature of EWMA and CUSUM charts.

Some notable work on detection performance of EWMA chart can be found in [12], [13] . Some have also used technique of distinct samples. Whereas, few have also used the run-rule technique [14]–[16]..

All these modifications of the EWMA chart are based on current observations and do not consider the past observations. To address these limitations, another modification to EWMA was proposed which uses a progressive mean [17]. However, in this modification, the control limits become wider as the time increases and hence the chance of detecting the out of control signals decreases.

The progressive mean control chart was proposed by [18], [19]. We executed the modified control chart in R to fix the value of Average Run Length (ARL0) and then find the values of value of Average Run Length ARL_1 for this prefixed value of ARL_0. As ARL_0 were very big in this case, so we reduced the limits by using a time function and searched a constant which not only fixed ARL_0 to a specific value but helped minimizing ARL_1. We used many functions and constants and then selected the optimum combination such that ARL_0 fixed to a particular value and ARL_1 minimized.

II. METHODOLOGY

Let have a particular quality variable to be monitored, say Y. assume that Y follows Normal distribution with target parameter $a_0 + \gamma b_0$, spread parameter b_0 and shift parameter γ. Let Y_t be the value of variable observed at each time period and $t = 1,2,3, \ldots m$. Let we have sample of size 1 i.e. $n = 1$. In this study from the normal distribution individual observations are taken. The Progressive Mean statistic is defined as

$$P_t = \frac{\sum_{i=1}^{t} Yi}{t}$$

The PM statistic accumulates, each upcoming observation and averages each time. P_t is an unbiased estimator a_0, that is the population mean and $\frac{b_0^2}{t}$, is variance. Where a_0 and b_0^2 are the mean and variance respectively, for in-control process. Three sigma control limits are defined as

$$UCL = Statistic + K \times Variance(Statitics)$$
$$LCL = Statistic - K \times Variance(Statitics)$$
$$UCL = a_0 + \frac{3b_0}{\sqrt{n}}$$
$$LCL = a_0 - \frac{3b_0}{\sqrt{n}}$$

If n=1

$$UCL = a_0 + 3b_0$$
$$LCL = a_0 + 3b_0$$

limits can be defined as for the progressive statistics

$$LCL_t = a_o - 3\frac{b_o}{\sqrt{t}}, \qquad CL = \mu_o, \quad UCL_t = a_o + 3\frac{b_o}{\sqrt{t}}$$

From above equation we can see that the limits vary over time as the time increases. This control chart uses all the information contained in the samples and hence the chance of detecting an out of control sample is zero with conventional control limits. So, we have some solutions. Either we change constant K of limits, or we change only function of time or else we can change both. Final one is the better solution for this problem. We can first change the function of time to have a finite Average Run Length. Then we change constant to settle limits to certain level to achieve desired ARL. The modified limits are as follows:

$$LCL_t = a_o - 3\frac{b_o}{\sqrt{t}}\left(\frac{K_0}{f(t)}\right), \qquad CL = \mu_o, \quad UCL_t = a_o + 3\frac{b_o}{\sqrt{t}}\left(\frac{k_0}{f(t)}\right)$$

Where $f(t)$ an function of t, and k_0 is controlling-constant the ARLs. ARLS are frequently used to measure in control charts, so is done in this study. The ARL distribution was calculated using Monte Carlo simulations. After finding distribution, we can easily calculate average of Run Lengths. For in control situations we have assumed $a_0 = 0$ and $b_0 = 1$, i.e. is standard normal distribution. The shift parameters is Δ. If $\Delta = 0$, its mean that process is in control, otherwise out of control. In this study we will try to enhance the performance and power of PM control chart using FIR. In this research study, we have done 10,000 simulation runs to evaluate different run length properties.

For the modified control limits, we used different options of $f(t)$ and search a suitable value to "k_0" and $t^{-0..20}$ is found to be suitable value for $f(t)$ in terms of optimizing the RL properties. For this optimal choice of $f(t)$ we have checked different values of constant k_0 which help fixing ARL_o. The ARL_0 is the average number of samples, we have to wait until a shift is detected for in control process. It is inversely proportional to α

$$ARL_0 = \frac{1}{\alpha}$$

The ARL_1 for the out of control process is the average number of samples, we have to wait until a shift is detected. It is inversely proportional to $1-\beta$

$$ARL_1 = \frac{1}{1-\beta}$$

III. RESULTS & DISCUSSION

The following results are obtained from above equations define in section (2.0) by using the method of monte Carlo simulation method:

Table 4.1. ARL_0 for the modified PM control chart with constant values of k_0 for the control limits

ARL_o	168	200	370	400	500
k_0	1.000	1.042	1.186	1.211	1.265

Table 4.2. Values of ARL of the proposed PM chart of different shifts

Prefixed ARL_0	γ									
	0	0.25	0.5	0.75	1	1.5	2	3	4	5
500	488.14	45.2251	17.029	10.1454	7.444	4.4204	3.114	1.8803	1.3847	1.1166
400	402.938	43.463	15.761	09.313	7.158	4.161	2.775	1.776	1.255	1.0647
370	370	15.441	09.055	09.144	7.0631	4.084	2.6931	1.621	1.1835	1.035
200	202.82	32.7144	13.5813	8.337	5.7804	3.4896	2.4914	1.5114	1.1570	1.0288
168	172.33	30.0684	11.1368	8.1221	5.351	3.3086	2.3436	1.4433	1.0987	1.0172

Table 4.3. Values of Standard deviation of Run Length (SDRL) for the proposed chart of different shifts

ARL_0	γ									
	0	0.25	0.5	0.75	1	1.5	2	3	4	5
500	977.233	36.891	11.858	6.0454	3.581	1.889	1.176	0.650	0.5062	0.3099
400	737.2564	35.771	11.443	5.7014	3.492	1.821	1.128	0.638	0.481	0.2771
200	388.5543	30.654	9.834	5.167	3.206	1.648	1.049	0.611	0.387	0.168
168	244.31	30.09	9.671	4.886	3.075	1.596	1.0287	0.587	0.354	0.154

Table 4.4. Percentiles (Q_j) using various shifts

ARL_0	Percentile	γ									
		0	0.25	0.5	0.75	1	1.5	2	3	4	5

500	P_{10}	23	10	6	4	3	2	1	1	1	1
	P_{25}	58.75	21	9	6	4	6	2	2	1	1
	P_{50}	175	36	13	9	6	5	3	2	2	1
	P_{75}	511	61	21	13	9	4	4	1	1	1
	P_{90}	1268.1	95	32	18	11	6	6	3	2	2
400	P_{10}	20.1	9	5	3	3	2	2	2	1	1
	P_{25}	49	18	8	5	4	3	2	1	2	1
	P_{50}	149	31	11	8	6	5	3	1	1	1
	P_{75}	430.25	59	21	12	8	3	4	2	2	1
	P_{90}	1038.1	89	31	16	10	7	3	3	2	1
200	P_{10}	10	6	3	3	2	2	1	2	1	1
	P_{25}	25	11	5	5	4	2	3	1	2	1
	P_{50}	70	24	10	6	5	4	2	2	1	1
	P_{75}	210	43	18	10	7	5	3	1	1	1
	P_{90}	499	75	27	15	8	6	4	2	2	1
168	P_{10}	10	4	4	2	2	1	2	1	1	1
	P_{25}	25	1	7	3	3	3	1	2	2	1
	P_{50}	59	21	10	6	5	3	2	1	1	1
	P_{75}	179	42	17	10	6	4	3	2	1	1
	P_{90}	422	70	28	14	11	5	4	1	2	1

Following the inspiration of Palm (1990), " Shmueli and Cohen (2003), [20]Antzoulakos and Rakitzis (2008), Riaz et al. (2010) and [16]Abbas *et al*., (2010), we have additionally reportable the standard deviations and percentile points of the Run Lengths to own a transparent plan concerning distribution of Run Length. The standard Deviations of Run Length (SDRL) and also the percentiles of run lengths (Qi for i=10, 25, 50, 75, 90) are given in Tables 4.3 and 4.4 correspondingly.

The similar results for different ARLo can be obtained simply. The standard errors for the results of Tables 4.1 – 4.4 remain less than 1.3%.

The important findings for our proposed PM chart as observed in the above investigation are given as:

1) The Modified Progressive Mean Control chart is admittedly sensible at smaller and moderate shift detection and additionally occupies an attractive place for detection of large shifts within the family of memory control charts (Table 4.2);
2) For fixed γ, the *SDRL* value of the proposed control chart increases with the increase in ARL_0 (Table 4.3);
3) The distribution of Run length (*RL)* of control chart is right skewed. (Table 4.4);
4) With an increase in γ ARL_1 decreases swiftly (Tables 4.1 & 4.2);

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5) If we apply the same design structure on $\bar{X}$ control chart with sample $n > 1$ then the performance of the proposed control chart becomes more attractive;
6) The proposed control chart is easy in terms of design structure and straightforward to implement than the present ones (like CUSUM and EWMA);
7) For consequences with higher powers of *t* (i.e. 0.5 and 0.75), ARL_1 performance of our proposed control chart becomes relatively poor;

A. Comparisons

The performance of our modified control chart with a number of its equivalents are compared, in terms of ARL. We executed the proposed control chart and compared the results with the existing structures of memory control chart such as the classical CUSUM, classical EWMA, and the FIR EWMA and etc.
The ARL_0 value for the proposed chart was fixed at 168, 200, 370, 400 and 500 for the effective comparisons with each of the existing chart. The comparisons of the modified PM control chart with each CUSUM and EWMA are given in the following subsections.

Proposed vs the classical CUSUM:

The classical CUSUM by [1] has been explained in Section 1.1. The *ARL* values of classical CUSUM specified by [4] are shows in Table 4.5. The comparison of the classical CUSUM with the modified PM control chart demonstrates that the proposed control chart for all intents and purposes overperform the classical CUSUM for all values of γ (Table 4.2 versus Table 4.5).

Table 4.5. The classical CUSUM scheme ARL values with k= 0.5

γ	0	0.25	0.5	0.75	1	1.5	2	2.5	3
***h* =4**	**168**	**74.2**	**26.6**	**13.3**	**8.38**	**4.75**	**3.34**	**2.62**	**2.19**
***h* = 5**	**465**	**139**	**38**	**17**	**10.4**	**5.75**	**4.01**	**3.11**	**2.57**

Proposed vs FIR EWMA:

The evaluation of FIR EWMA i.e. for head-start 25%, execution of the proposed control chart is better than FIR EWMA for each choice of λ. However, for head-start 50% the execution of FIR EWMA turns out to be superior to the proposed control chart for the larger shift with λ=0.1

Table 4.6. ARL values for the FIR EWMA scheme:

γ	% Head start	λ =0.1 L= 2.814	λ =0.25 L= 2.998	λ =0.5 L =3.071	λ =0.75 L= 3.087
0	25	487	491	497	498
	50	468	483	487	496
	25	28.3	46.5	87.8	140
0.5	50	24.2	43.6	86.1	139

1	25 50	8.75 6.87	10.1 8.79	16.9 15.9	30.2 29.7
2	25 2.72	3.57 50	3.11 2.5	3.29 2.87	4.33 4.09

B. Simulated Study:

This section presents the application strategy of the proposed control chart. Moreover, the EWMA and CUSUM charts are incorporated into the illustrative precedent to verify the prevalence of proposed approach applicable over these existing structures. For this reason, we generated two datasets each of 50 observations. In dataset I, starting 30 observations are generated from N (0,1) for in-control circumstance, and rest of the 20 from N(0.5,1) for wild circumstance having shift 0.25σ (small shift). Similarly, in dataset II initial 30 observations were created from N(0,1) for in-control situation and rest of 20 from N (1.5,1) for out of control situation having shift 1.5 σ (moderate shifts). EWMA statistic $\lambda = 0.25$ and CUSUM statistic C^{+} with $k = 0.5$ are calculated. "To fix the ARLo at 500. We used k_o=1.267 for modified PM control chart, L=2.998 for classical EWMA and h=5.09 for classical CUSUM. The graphical presentation of the proposed PM, EWMA", and CUSUM charts are exhibited in Figures 4.1, 4.2 and 4.3, respectively for dataset 1, and in Figures 4.4, 4.5 and 4.5, respectively for dataset II .

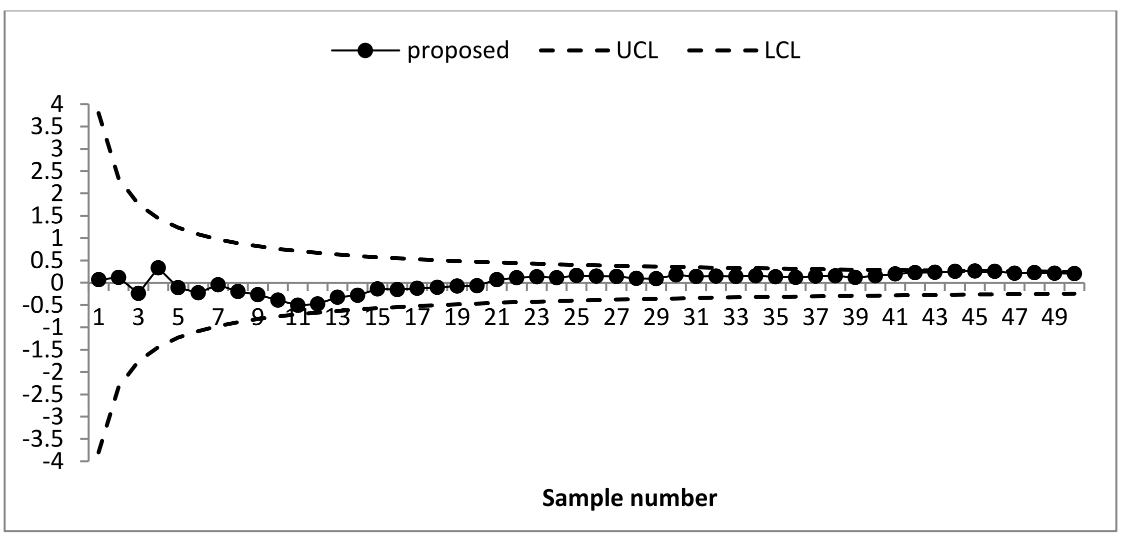

Figure 4.1. Graphical Presentation of the Modified PM Control Chart for Dataset I:

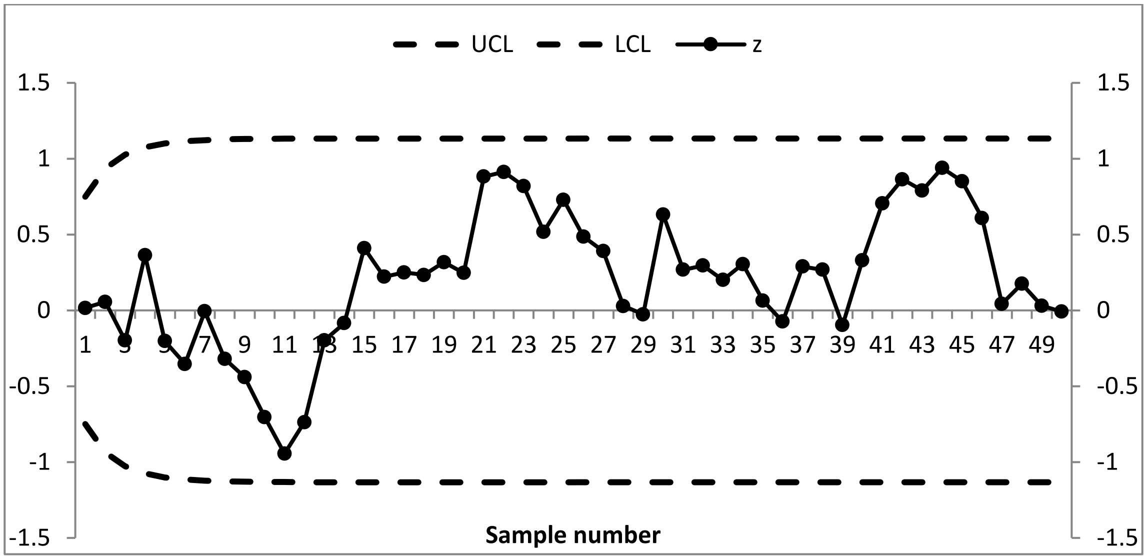

Figure 4.2. Graphical Presentation of the "Classical EWMA Chart" for Dataset- I:

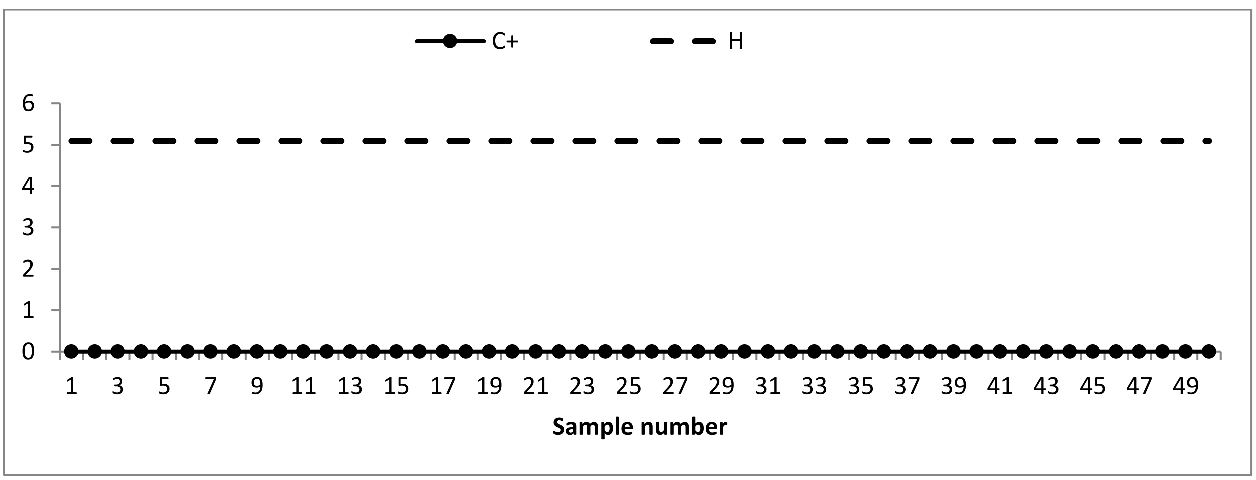

Figure 4.3. Graphical Presentation of the Classical CUSUM for Dataset I:

The modified control chart provides indications for an out of control process at sample # 43, 44, 45, 46, 47, 48, 49 and 50 as shown in Figure 4.1. It gives a total of 24 signals out of control. Figure 4.2 demonstrates that the classical EWMA control chart provides the out of control signal just at sample # 42, giving only 1 signal. Figure 4.3 portrays that the "classical CUSUM control chart gives none of the out of control signals". An upward shift happened after sample #20 that is recognized by the proposed chart rapidly before EWMA and CUSUM demonstrating the capacity of the modified control chart to rapidly distinguish smaller shifts in the whole process.

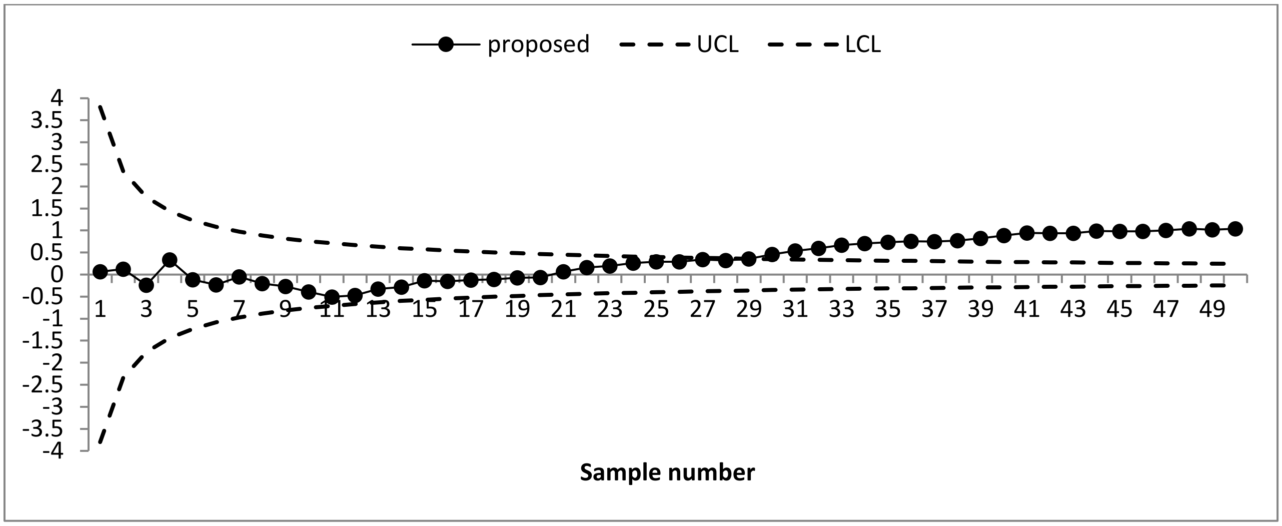

Figure 4.4. Graphical picture of the Modified PM Chart for Dataset II

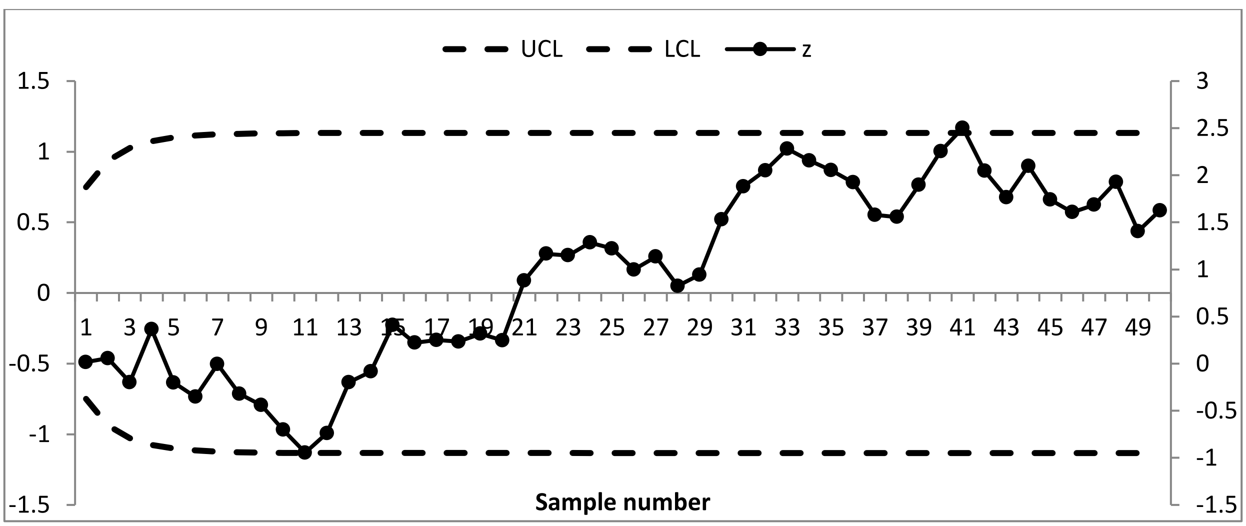

Figure 4.5. Graphical Presentation of the Classical EWMA for Dataset II

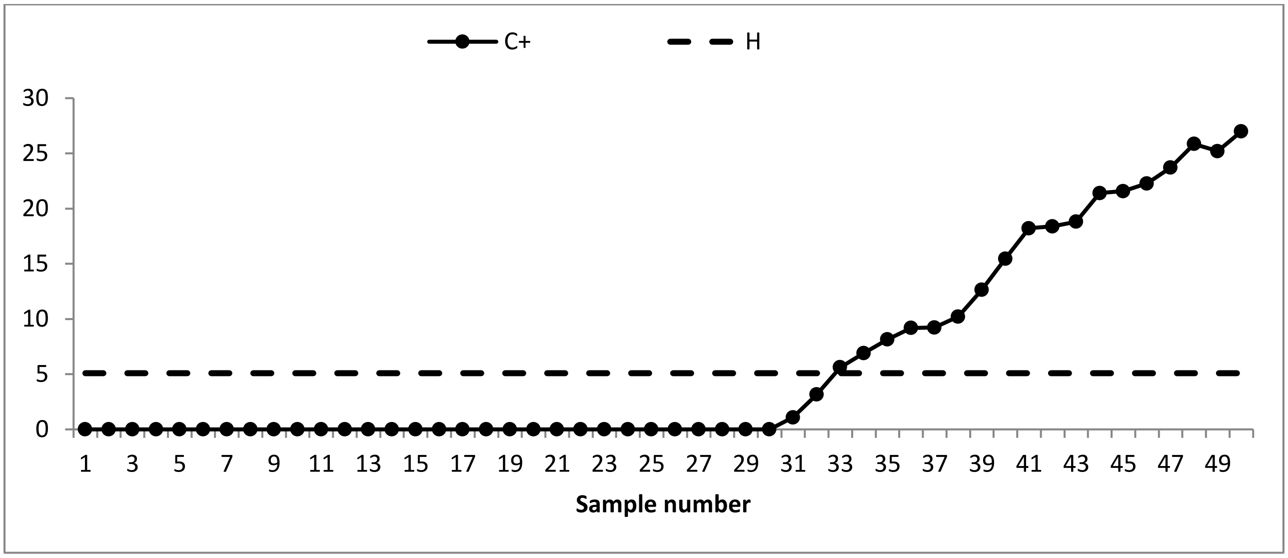

Figure 4.6. Graphical Picture of the Classical CUSUM for Dataset II

The situation is not massively different in dataset II where the proposed MPM chart recognizes the shift at sample # 27, 28, 29, 30, 31, 32, 33… 50 giving 24 an out of control signals (Figure 4.4). Classical EWMA identified the shift at sample # 27, 28, 29 and 30 giving 4 out of control signals (Figure 4.5) whereas classical CUSUM and EWMA chart signaled at the same points (Figure 4.6). In both smaller and larger shifts, we have seen that the suggested control chart identifies the shift rapidly before the others. In addition, the quantity of signals given by modified control chart, is likewise more prominent than the exemplary ones.

IV. CONCLUSIONS

In this we executed the Modified control chart with the different constant values. The results show a higher power for small, moderated, and larger shifts than the existing charts However, for a higher power of *t* (time period), ARL_1 performance of our proposed control chart becomes relatively poor than the existing structures.

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