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Extending the Stochastic Approach to Paasches Price Index Numbers

Prof. Dr. S.M. Aqil Burney[1] and Arfa Maqsood[2]

ABSTRACT

In recent times, the stochastic approach has received enormous attention to estimate the rate of inflation. The attraction of this approach is to provide not only the estimate of inflation rate, but also its standard error. In this paper, we extend the stochastic approach to derive the Paasches price index number and its standard error. We present an illustration to Laspeyres index number using consumer price data of Pakistan covering the period from July 2002 to June 2011.

1. INTRODUCTION

There are two foremost approaches to index number theory. The first is the functional index number approach, with a long history going back to the work of Konus (1924), provides the basic framework for measuring changes in consumer prices over time. Later on this method has been augmented by the effort of Samuelson and Swamy (1974), and Diewert (1981).

The other methodology, called 'atomistic' or 'stochastic' approach by Frisch (1936), associated with Stanley Jevons and Francis Edgeworth [see Frisch (1936)]. In the last two decades, this approach has been familiar and attracting great consideration to measure the index numbers. The desirable feature of this approach is to provide not only the point estimate of inflation rate, but its standard error, so that the confidence intervals are possible to construct for the true rate of inflation.

[1] Dr. S. M. Aqil Burney is Meritorious Professor at the Department of Computer Science, University of Karachi and is Professor at the Computer Science Department IoBM burney@uok.edu.pk
[2.] Arfa Maqsood is Assistant Professor at the Department of Statistics, University of Karachi. amaqsood@uok.edu.pk

Moreover, it is a way of viewing index number in which uncertainty and statistical ideas play a vital role, rather than just providing a single number, it determines the whole distribution of inflation rate. It has widely been used by many authors to derive the underlying rate of inflation and its standard error. Clements and Izan (1981, 1987) have immensely contributed their part to develop the divisia index number. They extended the approach to index number by allowing for sustained commodity changes in relative prices. Further motivation in this area, affiliated with the work of Selvanathan (1989, 1991, 1993, 2003), Selvanathan and Rao (1992, 1994), Clements and Selvanathan (2007), and Selvanathan and Selvanathan (2006). In the context of Pakistan, Burney (1995) has reviewed the measures of central tendency in stock exchange price index. Burney and Maqsood estimated the inflation rate using various time series model.

The objective of this study is to counter the Keynes's criticism using a commodity-specific component to the model of Selvanathan and develop the maximum likelihood estimator of Paasches index number and its standard error.

Selvanathan (1991) used the regression approach to derive the standard errors for the Laspeyres and Paasches index numbers. Selvanathan (1993) answered the criticism by extending the approach to Laspeyres index number using a commodity dummy variable. Selvanathan (1991) uses the regression of expenditure on commodity i in current period, $p_{it}q_{it}$, on current period consumption, valued at base period, $p_{io}q_{it}$, i.e.

$$p_{it}q_{it} = p_{io}q_{it} + \varepsilon_{it} \qquad ; i = 1,2,\ldots,n \,; t = 1,2,\ldots,T \tag{1}$$

Where n is the number of commodities and ε_{it} is the zero-mean random component. The GLS estimator of α_t is identical to the usual Paasches price index $I_{ot} = \sum p_{it}q_{it} / \sum p_{io}q_{it}$. Dividing eq. (1) by $p_{io}q_{it}$, we get

$$P_{it}^{o} = \alpha_t + \xi_{it} \qquad ; \ i = 1, 2, \ldots, n \tag{2}$$

We note that $\xi_{it} = P_{it}^{o} - \alpha_t$, the change in the ith relative prices, and the mathematical expectation of random component becomes

$$E(\xi_{it}) = E(P_{it}^{o} - \alpha_t) = 0 \qquad ; i = 1, 2, \ldots, n$$

Which indicate the expected value of relative price change is zero for all i, this is undoubtedly weakness of the model and recognized by Keynes (1930). We, therefore, extend the work presented in Selvanathan (1991) using the stochastic approach to index number theory.

In section 2, we introduce the extended version of model (2) that allows systematic changes in relative prices. Afterwards, we derive the Paasches index number using the maximum likelihood estimation method. We, then, present an illustrative application to estimate the Laspeyres price inflation rate and its standard error in the context of Pakistan. Concluding remarks are recapitulated in section 4.

2. THE NEW EXTENDED MODEL

We extent the model by incorporating the commodity-specific component to permit sustained changes in relative prices. Each price relative $\mathrm{P}_{\mathrm{it}}^{\mathrm{o}} = \mathrm{p}_{\mathrm{it}}/\mathrm{p}_{\mathrm{io}}$ is written, as the sum of the common trend in all prices, α_{t}, a systematic component β_{i}, and a random error term, i.e.:

$$\mathrm{P}_{\mathrm{it}}^{\mathrm{o}} = \alpha_t + \beta_i + \xi_{it} \qquad ; \; i = 1, 2, \ldots, \mathrm{n} \; ; \; \mathrm{t} = 1, 2, \ldots, \mathrm{T} \tag{3}$$

Here ζ_{it} is assumed to be normally distributed with mean $E(\xi_{it}) = 0$ and covariance $Cov(\xi_{it}\xi_{jt}) = \dfrac{\eta_t^2}{w_{it}}\delta_{ij}$. Where δ_{ij} is kronecker delta and $w_{it} = \mathrm{p}_{\mathrm{it}}\mathrm{q}_{\mathrm{it}} / \sum \mathrm{p}_{\mathrm{it}}\mathrm{q}_{\mathrm{it}}$ is the budget share of commodity i in period t. Now rearranging eq. (3) and taking the mathematical expectation, we obtain the change in ith relative price as $E(\mathrm{P}_{\mathrm{it}}^{\mathrm{o}} - \alpha_t) = \beta_i$.

Model (3) is not identified, as an increase in α_t for each t by any number k and a lowering of β_i for each i by the same k does not affect the right side of equation (3).

To identify the model, we impose the constraint

$$\sum_{i=1}^{n} w_{it}\beta_i = 0 \tag{4}$$

This has the simple interpretation that a budget share weighted average of the systematic component of the relative price changes is 0.

3. THE MLE OF PAASCHES INDEX NUMBER

We apply the maximum likelihood estimation method to estimate the parameters of model (3) subject to constraint (4). This model can be written using the constraint equation as

$$P_{it}^{o} = \begin{cases} \alpha_t + \beta_i + \xi_{it}, & \text{i} = 1,2,\ldots,\text{n - 1} \\ \alpha_t + \sum_{i=1}^{n-1}\left(-\frac{w_{jt}}{w_{nt}}\right)\beta_j + \xi_{nt}, & \text{i} = \text{n} \end{cases}$$

The matrix formulation of this model is

$$\begin{bmatrix} p_{1t}^{o} \\ p_{2t}^{o} \\ . \\ . \\ p_{n-1,t}^{o} \\ p_{nt}^{o} \end{bmatrix} = \begin{bmatrix} I & \iota & o & . & . & o \\ I & o & \iota & . & . & o \\ . & . & . & . & . & . \\ . & . & . & . & . & . \\ I & o & o & . & . & \iota \\ I & & A & & & \end{bmatrix} \begin{bmatrix} \alpha_1 \\ . \\ . \\ \alpha_T \\ \beta_1 \\ . \\ . \\ \beta_{n-1} \end{bmatrix} + \begin{bmatrix} \xi_{1t} \\ \xi_{2t} \\ . \\ . \\ \xi_{n-1,t} \\ \xi_{nt} \end{bmatrix}$$

Where I is an identity matrix, ι and o are the column vectors of one and zeroes respectively, and

$A = \begin{bmatrix} A_1 & A_2 & . & . & A_{n-1} \end{bmatrix}$ and $A_i = \begin{bmatrix} A_{i1} & A_{i2} & . & . & A_{i,n-1} \end{bmatrix}$

Using an obvious notation, we write the model as

$$P_{(nT\times1)} = X_{nT\times(T+n-1)}\gamma_{(T+n-1)\times1} + \xi_{(nT\times1)} \tag{5}$$

ζ_t are assumed to have multinormal distribution with mean zero and variance-covariance matrix

$$Cov(\xi_{it},\xi_{js}) = \eta_t^2 \begin{bmatrix} 1/w_{1t} & 0 & . & . & 0 \\ 0 & 1/w_{2t} & . & . & 0 \\ . & . & & & . \\ . & . & & & . \\ 0 & 0 & . & . & 1/w_{nt} \end{bmatrix} = \eta_t^2 W_t^{-1}$$

The log-likelihood function of model (5) is

$$LogL(\eta_1^2,..,\eta_{\mathrm{T}}^2,\gamma) = c - \frac{n}{2}\sum \log\eta_t^2 - \sum\frac{1}{2\eta_t^2}(P - X\gamma)' W_t (P - X\gamma) \tag{6}$$

Where c is a constant, taking the first derivative of eq. (6) to obtain the estimates of parameter vector γ;

Table 1: Estimates of Laspeyres index numbers and their standard errors: Pakistan (July 2001-June 2002 = 1.0)

S.Nos.	Time Period	$\hat{\alpha}_t$	SE ($\hat{\alpha}_t$)	True index	S.Nos.	Time Period	$\hat{\alpha}_t$	SE ($\hat{\alpha}_t$)	True index
1	Jul-02	1.031	0.143	1.060	55	Jan-07	1.363	0.097	1.410
2	Aug-02	1.027	0.144	1.064	56	Feb-07	1.367	0.087	1.425
3	Sep-02	1.029	0.146	1.066	57	Mar-07	1.360	0.081	1.432
4	Oct-02	1.024	0.143	1.067	58	Apr-07	1.377	0.101	1.436
5	Nov-02	1.021	0.143	1.067	59	May-07	1.378	0.109	1.449
6	Dec-02	1.023	0.140	1.064	60	Jun-07	1.376	0.095	1.452
7	Jan-03	1.024	0.140	1.066	61	Jul-07	1.434	0.070	1.467
8	Feb-03	1.024	0.141	1.071	62	Aug-07	1.442	0.073	1.486
9	Mar-03	1.017	0.145	1.071	63	Sep-07	1.469	0.088	1.518
10	Apr-03	1.027	0.152	1.075	64	Oct-07	1.481	0.100	1.537
11	May-03	1.019	0.149	1.071	65	Nov-07	1.478	0.062	1.539
12	Jun-03	1.011	0.148	1.069	66	Dec-07	1.496	0.063	1.548
13	Jul-03	1.051	0.143	1.075	67	Jan-08	1.523	0.083	1.577
14	Aug-03	1.050	0.142	1.082	68	Feb-08	1.518	0.097	1.585
15	Sep-03	1.054	0.139	1.089	69	Mar-08	1.550	0.155	1.634
16	Oct-03	1.066	0.132	1.105	70	Apr-08	1.612	0.135	1.683
17	Nov-03	1.070	0.130	1.112	71	May-08	1.647	0.152	1.729
18	Dec-03	1.085	0.122	1.122	72	Jun-08	1.668	0.152	1.765
19	Jan-04	1.084	0.125	1.121	73	Jul-08	1.785	0.181	1.824
20	Feb-04	1.074	0.125	1.117	74	Aug-08	1.805	0.171	1.863
21	Mar-04	1.076	0.130	1.128	75	Sep-08	1.817	0.168	1.881
22	Apr-04	1.092	0.132	1.139	76	Oct-08	1.849	0.171	1.921
23	May-04	1.095	0.125	1.147	77	Nov-08	1.841	0.140	1.919
24	Jun-04	1.098	0.119	1.160	78	Dec-08	1.845	0.138	1.909
25	Jul-04	1.152	0.109	1.176	79	Jan-09	1.836	0.126	1.901
26	Aug-04	1.149	0.109	1.182	80	Feb-09	1.844	0.119	1.919
27	Sep-04	1.151	0.105	1.187	81	Mar-09	1.849	0.124	1.945
28	Oct-04	1.159	0.102	1.201	82	Apr-09	1.891	0.140	1.973
29	Nov-04	1.166	0.101	1.214	83	May-09	1.887	0.144	1.977
30	Dec-04	1.165	0.097	1.204	84	Jun-09	1.893	0.131	1.997
31	Jan-05	1.176	0.106	1.216	85	Jul-09	1.983	0.164	2.028
32	Feb-05	1.180	0.098	1.228	86	Aug-09	1.994	0.146	2.062
33	Mar-05	1.183	0.103	1.244	87	Sep-09	2.000	0.151	2.071
34	Apr-05	1.207	0.104	1.265	88	Oct-09	2.011	0.152	2.091
35	May-05	1.198	0.100	1.260	89	Nov-09	2.035	0.158	2.120
36	Jun-05	1.195	0.100	1.261	90	Dec-09	2.041	0.168	2.110
37	Jul-05	1.250	0.090	1.281	91	Jan-10	2.087	0.182	2.161
38	Aug-05	1.242	0.091	1.282	92	Feb-10	2.087	0.180	2.169
39	Sep-05	1.246	0.092	1.288	93	Mar-10	2.094	0.185	2.197
40	Oct-05	1.252	0.093	1.300	94	Apr-10	2.145	0.188	2.234
41	Nov-05	1.257	0.091	1.310	95	May-10	2.127	0.172	2.236
42	Dec-05	1.262	0.090	1.307	96	Jun-10	2.127	0.172	2.250
43	Jan-06	1.277	0.108	1.322	97	Jul-10	2.222	0.193	2.278

44	Feb-06	1.275	0.104	1.327	98	Aug-10	2.253	0.223	2.335
45	Mar-06	1.268	0.109	1.330	99	Sep-10	2.314	0.311	2.397
46	Apr-06	1.292	0.119	1.343	100	Oct-10	2.319	0.260	2.412
47	May-06	1.286	0.102	1.349	101	Nov-10	2.357	0.332	2.448
48	Jun-06	1.286	0.102	1.357	102	Dec-10	2.357	0.286	2.436
49	Jul-06	1.344	0.088	1.379	103	Jan-11	2.385	0.293	2.468
50	Aug-06	1.351	0.099	1.396	104	Feb-11	2.349	0.237	2.449
51	Sep-06	1.353	0.086	1.401	105	Mar-11	2.363	0.250	2.486
52	Oct-06	1.354	0.094	1.406	106	Apr-11	2.422	0.281	2.526
53	Nov-06	1.364	0.115	1.416	107	May-11	2.403	0.269	2.532
54	Dec-06	1.376	0.132	1.423	108	Jun-11	2.401	0.254	2.546

Note: True indices are calculated officially by PBS using July 2000-June 2001 base period

$$\hat{\gamma} = \left(\sum \frac{1}{\eta_t^2} X'W_t X \right)^{-1} \left(\sum \frac{1}{\eta_t^2} X'W_t P \right)$$

This simplifies to

$$\hat{\alpha}_t = \sum_{i=1}^{n} w_{it}\, p_{it}^o \text{ , and } \hat{\beta}_i = \sum_{t=1}^{T} \phi_t \left(p_{it}^o - \hat{\alpha}_t \right) \qquad (7)$$

An appendix may be available from the author upon request.

4. AN ILLUSTRATION

In Pakistan, the prices are compared with the specific period that is fixed for usually every ten years. Hence the rates of inflation based on Laspeyres price index numbers are calculated for several time periods. To work with consumer price data of Pakistan, we have limitations to compute the Paasches price index number. We present an illustrative application to calculate the Laspeyres price index numbers (with base period Jul 2001-June 2002) and their standard errors using the average consumer prices of overall Pakistan, obtained from the Monthly Bulletin of Statistics, published by Population Bureau of Statistics (PBS).

The estimates of Laspeyres price index numbers and their standard errors are presented in table 1 for the period July 2002 to June 2011. The genera price level has remarkably been doubled in July 2009 relative to July 2001, in almost eight years, with a standard error of13 percent. The point estimates of $\hat{\alpha}_t$ are approximately close to the indices, calculated by PBS. This negligible dissimilarity is due to use of two different base periods. Table 2 shows the relative prices of food & beverages, fuel & lighting, and transport & communication slightly increase by about

7.3, 2, and 3.5 respectively. All other seven commodity groups exhibit a decline in relative prices. The estimates are highly significant for the recreation & entertainment, and medicare groups with respective standard errors 0.5, and 0.425.

Table 2: Estimates of commodity-specific components for commodity groups

Group No.	Commodity Groups	β_i	SE (β_i)
1	FOOD & BEVERAGES.	0.073	0.199
2	APPAREL, TEXTILE & FOOTWEAR.	-0.158	0.298
3	HOUSE RENT INDEX	-0.014	0.021
4	FUEL AND LIGHTING.	0.020	0.163
5	H.HOLD.FURNITURE & EQUIPMENT	-0.180	0.416
6	TRANSPORT & COMMUNICATION.	0.035	0.275
7	RECREATION & ENTERTAINMENT.	-0.336	0.500
8	EDUCATION.	-0.210	0.299
9	CLEANING LAUNDRY & PER.APPEA	-0.128	0.281
10	MEDICARE.	-0.308	0.425

The scatter diagram of estimated index numbers against its corresponding standard errors is illustrated in figure 1, indicating a positive association between them. In other words, the higher index number is not easy to estimate precisely in absolute mode that is standard error of index tends to rise. The rise in index number is higher than the increase in standard error, making estimation of index number is quite challenging when the standard error is higher. The straight line is the least square regression line, suggesting the direct relationship. In figure 2, we plot the estimates of index number along with its 95% confidence bands constructed using the normal distribution $\hat{\alpha}_t \pm 1.96\, SE(\hat{\alpha}_t)$. Index number is continuously rising over the years, which is as expected, since CPI has an increasing pattern in nature. The width of confidence bands at the starting months and in the mid of 2010.

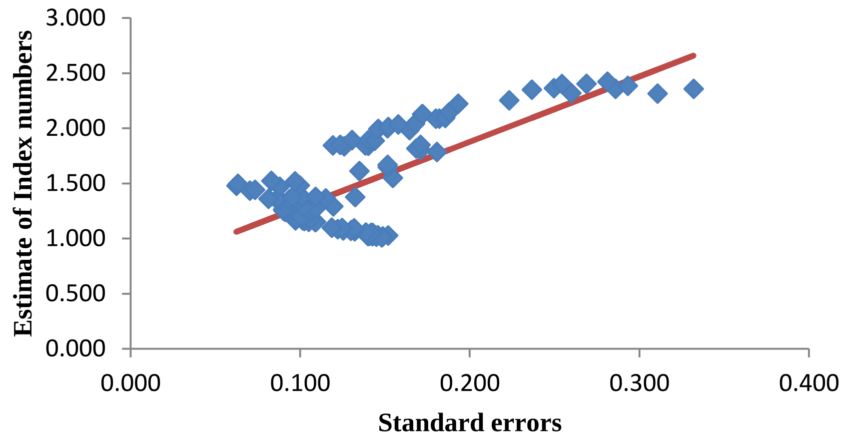

Figure 1: Index numbers and their standard errors

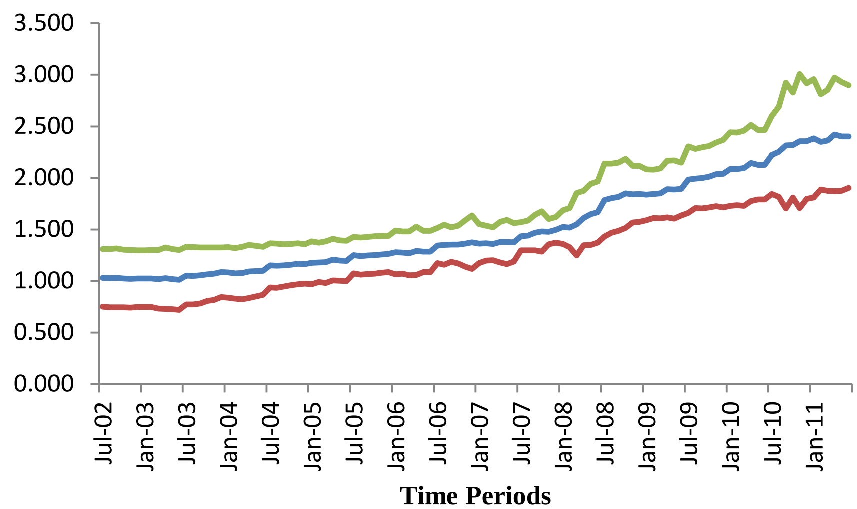

Figure 2: Index numbers and their 95% confidence bands

5. CONCLUSION

According to Keynes's criticism the stochastic approach used by Selvanathan (1991) did not allow the systematic changes in relative prices. First, we extended the stochastic approach to index number by means of incorporating a commodity-specific component to the equation of

Paasches index number. Second, we used the technique of maximum likelihood estimation to derive the estimate of Paasches index number and its standard error. Lastly, we presented an illustrative application to compute the Laspeyres price index number and its standard error, derived by Selvanathan (1993), with reference to Pakistan data.

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